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1. * $x^3-22x-24=0$. Here $p=\frac{22}{3}$, $q=12$.

$$\text{Then } x=\sqrt[3]{12+\tfrac{26}{3}\sqrt{(-\tfrac{10}{3})}}+\sqrt[3]{12-\tfrac{26}{3}\sqrt{(-\tfrac{10}{3})}}.$$

Now, $a=m^3+3mn^2=12=-2^3+20$. $3mn^2=20$, $n=\sqrt{-\frac{10}{3}}$.

$\therefore$ $-2\pm\sqrt{-\frac{10}{3}}$ are the roots. Then $x=(-2+\sqrt{-\frac{10}{3}})+(-2-\sqrt{-\frac{10}{3}})=-4$.

2. $x^3-8x^2+19x-12=0$. Put $x=y+\frac{8}{3}$.

Then $y^3-\frac{7}{3}y+\frac{20}{27}=0$, where $p=\frac{7}{9}$, $q=-\frac{10}{27}$.

$$\text{Then } y=\sqrt[3]{-\tfrac{10}{27}+\sqrt{(-\tfrac{243}{729})}}+\sqrt[3]{-\tfrac{10}{27}-\sqrt{(-\tfrac{243}{729})}}$$

$$=\sqrt[3]{-\tfrac{10}{27}+\tfrac{1}{3}\sqrt{-3}}+\sqrt[3]{-\tfrac{10}{27}-\tfrac{1}{3}\sqrt{-3}}$$

Here $a=m^3+3mn^2=-\frac{10}{27}=(\frac{2}{3})^3-\frac{18}{27}$. Hence $n=\sqrt{-\frac{1}{3}}=\frac{1}{3}\sqrt{-3}$.

$\therefore$ $y=(\frac{2}{3}+\frac{1}{3}\sqrt{-3})+(\frac{2}{3}-\frac{1}{3}\sqrt{-3})=\frac{4}{3}$. Then $x=\frac{4}{3}+\frac{8}{3}=4$.

3. $x^3-12x^2+41x-42=0$. Put $x=y+4$.

Then $y^3-7y=6$, where $p=\frac{7}{3}$, $q=3$. Then $y=\sqrt[3]{3+\frac{10}{9}\sqrt{-3}}+\sqrt[3]{3-\frac{10}{9}\sqrt{-3}}$.

Here $a=3=(\frac{3}{2})^3-\frac{3}{8}$. $\therefore$ $n=\frac{1}{6}\sqrt{-3}$.

$\therefore$ $y=(\frac{3}{2}+\frac{1}{6}\sqrt{-3})+(\frac{3}{2}-\frac{1}{6}\sqrt{-3})=3$, and $x=3+4=7$.

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* Maynard in his Key to Bonnycastle's Introduction, page 78, says this "can only be resolved by a table of sines, or by infinite series."

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THEOREM 16 OF LOBATSCHEWSKY'S THEORY OF PARALLELS.

By JOHN N. LYLE, Ph. D., Professor of Natural Sciences, Westminster College, Fulton, Missouri

Says Lobatschewsky in his Theorem 16—"All straight lines which in a plane go out from a point can, with reference to a given straight line in the same plane, be divided into two classes—into *cutting* and *not-cutting*. The *boundary lines* of the one and the other class of those lines will be called *parallel to the given line.*

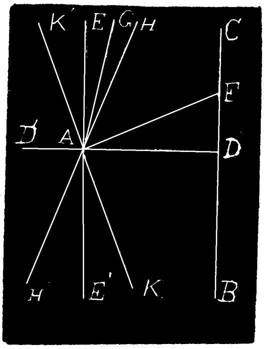

Fig. 1.

From the point A (Fig. 1) let fall upon the line BC the perpendicular AD, to which again draw the perpendicular AE. In the right angle EAD either will all all straight lines which go out from the point A meet the line DC, as for example AF, or some of them, like the perpendicular AE, will not meet the line DC. In the uncertainty whether the perpendicular AE is the only line which does not meet DC, we will assume it may be possible that there are still other lines, for example AG, which do not cut DC, how far soever they may be prolonged. In passing over from the cutting lines, as AF, to the not-cutting lines, as AG, we must come upon a line AH, parallel to DC, a boundary line, upon one side of which all lines AG are such as do not meet the line DC, while upon the other side every straight line AF cuts the line DC.

The angle HAD between the parallel HA and the perpendicular AD is called the parallel angle (angle of parallelism), which we will here designate by $\Pi(p)$ for $AD=p$."

Does Lobatschewsky class his boundary line AH among the *cutting* or the *not-cutting* lines? Evidently among the cutting lines, for under Theorem 33, referring to his equation $S'=se^{-x}$, he says—"We may here remark, that $S'=0$ for $x=\infty$, hence not only does the distance between two parallels decrease (Theorem 24), but with the pralongation of the parallels towards the side of the parallelism this at last wholly vanishes."

Agreeably to this assumption of Lobatschewsky let y be the point in space at which the decreasing distance between his parallel lines AH and DC wholly vanishes. According to Euclid's postulate 2 the terminated line Dy may be extended beyond the point y. If this is not permitted, Euclid's postulate 2 would be discredited in Lobatschewsky's geometry. Assume that Euclid's postulates hold everywhere in space. On the basis of that assumption we have the authority to locate any point as z beyond y on Dy extended. Then the point z is within the angle yAE. From z draw a straight line to the point A. This must be permitted, otherwise postulate 1 would be discredited in Lobatschewsky's geometry. Since z is within the angle yAE the straight line Az must fall between Ay and AE.

But since by Lobatschewsky's hypothesis no straight line between AHy and AE can meet DC produced, the line Az must fall between Ay and AD. That is, Az must lie on both sides of Ay at the same time. Says W. K. Clifford, an enthusiastic admirer of Lobatschewsky's Imaginary Geometry—"but the way things come out of one onother is quite lovely."

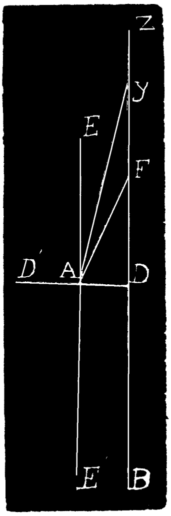

Fig. 2.

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SOME FALLACIES OF AN ANGLE TRISECTOR.

By LEONARD E. DICKSON, M. A., Fellow in Mathematics, The University of Chicago.

Since 1860, Mr. L. S. Benson of New York City has labored to throw